

Magic nuclei in superheavy valley

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Abstract

An extensive theoretical search for the proton magic number in the superheavy valley beyond $Z = 82$ and corresponding neutron magic number after $N = 126$ is carried out. For this we scanned a wide range of elements $Z = 112 - 130$ and their isotopes. The well established non-relativistic Skryme-Hartree-Fock and Relativistic Mean Field formalisms with various force parameters are used. Based on the calculated systematics of pairing gap, two neutron separation energy and the shell correction energy for these nuclei, we find $Z = 120$ as the next proton magic and $N = 172, 182/184, 208$ and 258 the subsequent neutron magic numbers.

Keywords:

After the discovery of artificial transmutation of elements by Sir Ernest Rutherford in 1919 [1], the search for new elements is an important issue in nuclear science. The existence of elements beyond the last heaviest naturally occurring ^{238}U , i.e., the discovery of Neptunium, Plutonium and other 14 elements (transuranium elements), which make a separate block in Mendeleev's periodic table was a revolution in the Nuclear Chemistry. This enhancement in the periodic table raises a few questions in our mind:

- Whether there is a limited number of elements that can co-exist either in nature or can be produced from artificial synthesis by using modern technique ?
- What is the maximum number of protons and neutrons that of a nucleus ?
- What is the next double shell closure nucleus beyond ^{208}Pb ?

To answer these questions, first we have to understand the agent which is responsible to rescue the nucleus against Coulomb repulsion. The obvious reply is the shell energy, which stabilises the nucleus against Coulomb disintegration [2]. Many theoretical models, like the macroscopic-microscopic (MM) calculations to explain involve some prior knowledge of den-

sities, single-particle potentials and other bulk properties which may accumulate serious error in the largely extrapolated mass region of interest. They predict the magic shells at $Z = 114$ and $N = 184$ [3, 4, 5, 6] which could have surprisingly long life time even of the order of a million years [7, 8, 9, 5, 10]. Some other such predictions of shell-closure for the superheavy region within the relativistic and non-relativistic theories depend mostly on the force parameters [11, 12].

Experimentally, till now, the quest for superheavy nuclei has been dramatically rejuvenated in recent years owing to the emergence of hot and cold fusion reactions. In cold fusion reactions involving a doubly magic spherical target and a deformed projectile were used by GSI [13, 14, 15, 16, 17] to produce heavy elements upto $Z = 110-112$. In hot fusion evaporation reactions with a deformed transuranium target and a doubly magic spherical projectile were used in the synthesis of superheavy nuclei $Z = 112-118$ at Dubna [18, 19, 20, 21, 22, 23, 24]. At the production time of $Z = 112$ nucleus at GSI the fusion cross section was extremely small (1pb), which led to the conclusion that reaching still heavier elements will be very difficult. At this time, the emergence of hot fusion reactions using ^{48}Ca projectiles at Dubna has dramatically changed the situation and nuclei with $Z = 114-118$ were synthesized and also observed their α -decay as well as termi-

nating spontaneous fission events. It is observed that $Z = 115$ – 117 nuclei have long α -decay chains contrast to the short chains of $Z = 114$ – 118 . Moreover, the life times of the superheavy nuclei with $Z = 110$ – 112 are in milliseconds and microseconds whereas the life time of $Z = 114$ – 118 up to 30 s. This pronounced increase in life times for these heavier nuclei has provided great encouragement to search the magic number somewhere beyond $Z = 114$. Moreover, it is also an interesting and important question for the recent experimental discovery [25, 26, 27] say chemical method of $Z = 122$ from the natural $^{211,213,217,218}\text{Th}$ which have long lived superdeformed (SD) and/or hyperdeformed (HD) isomeric states 16 to 22 orders of magnitude longer than their corresponding ground-state (half-life of $^{292}122$ is $t_{1/2} \geq 10^8$ years).

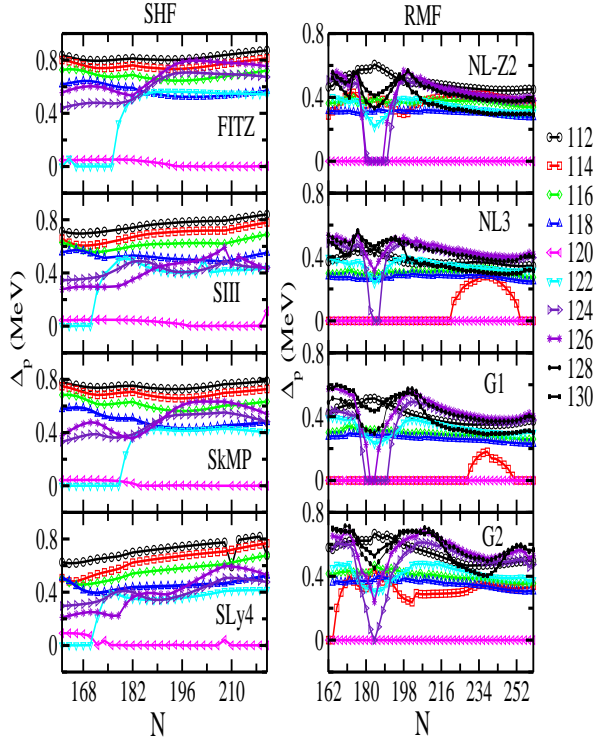


Figure 1: The proton average pairing gap Δ_p for $Z=112$ – 126 with $N=162$ – 220 and $Z=112$ – 130 with $N=162$ – 260 .

In this letter, our aim is to look for the next double closed nucleus beyond ^{208}Pb which may be a possible candidate for the experimentalists to look for. For this, we have used two well-defined but distinct approaches (i) non-relativistic Skryme-Hartree-Fock (SHF) with FITZ, SIII, SkMP and SLy4 interactions [28, 29] (ii)

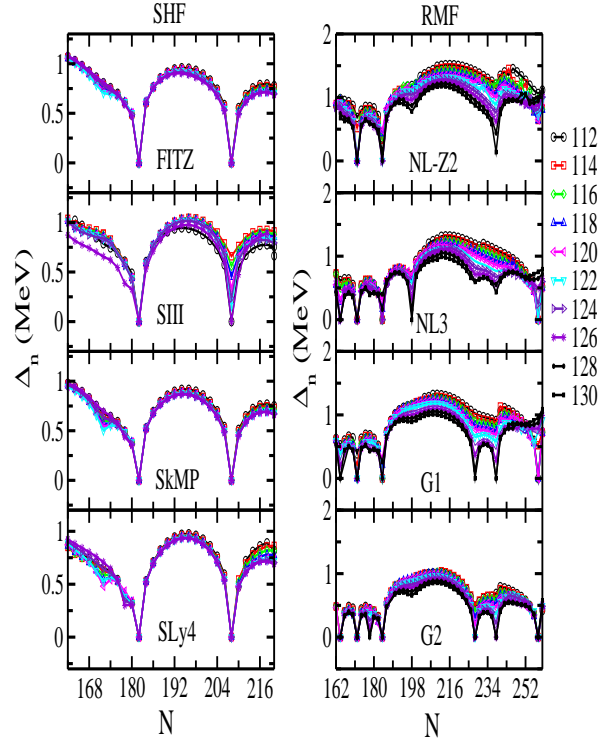


Figure 2: Same as FIG.1 but for neutron average pairing gap Δ_n .

Relativistic Mean Field (RMF) formalism [30, 31] with NL3, G1, G2 and NL-Z2 parameter sets. These models have been successfully applied in the description of nuclear structure phenomena both in β -stable and β -unstable regions throughout the periodic chart. The constant strength scheme is adopted to take care of pairing correlation [32] and evaluated the pairing gaps Δ_n and Δ_p for neutron and proton respectively from the celebrity BCS equations [33].

We scanned a wide range of nuclei starting from the proton-rich to the neutron-rich region in the superheavy valley ($Z=112$ to $Z=130$). It is well understood and settled that the properties of a magic number for a nuclear system has the following characteristics:

- The average pairing gap for proton Δ_p and neutron Δ_n at the magic number is minimum.
- The binding energy per particle is maximum compared to the neighboring one, i.e. there must be a sudden decrease (jump) in two neutron (or two proton) separation energy S_{2n} just after the magic number in an isotopic or isotonic chain.

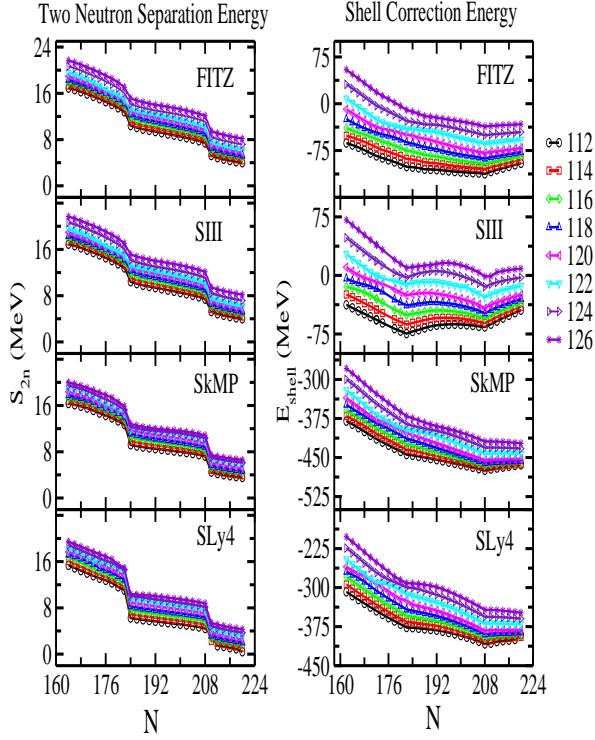


Figure 3: The two neutron separation energy S_{2n} and the shell correction energy E_{shell} for $Z=112-126$ and $N=162-220$ in the framework of SHF theory

- At the magic number, the shell correction energy E_{shell} is maximum negative. In other words, a pronounced energy gap in the single-particle levels $\epsilon_{n,p}$ appears at the magic number.

We focus on the shell closure properties in the super-heavy valley based on the above three important observables and identify the magic proton and neutron numbers.

The average pairing gap for proton Δ_p and for neutron Δ_n are the representative of strength of the pairing correlations. The curves for Δ_p are displayed in FIG. 1 obtained by SHF and RMF with FITZ, SIII, SLy4, SkMP and NL3, NL-Z2, G1, G2 force parameterizations. If we investigate the figure carefully, it is clear that the value of Δ_p almost zero for the whole $Z=120$ isotopic chain in both the theoretical approaches. A similar Δ_p is observed for few cases of $Z=124$ and $Z=114$ isotopes.

To predict the corresponding neutron shell closure of the magic $Z=120$, we have estimated the neutron pairing gap Δ_n for all elements $Z=112-130$ with their corresponding isotopic chain. As a result of this, the cal-

Table 1: Single-particle levels for neutron ϵ_n (MeV) for $^{302}120$ in SHF(SLy4 and FITZ) and $^{304}120$ in RMF (NL3 and G2).

Orbit	neutron (ϵ_p)			
	SLy4	FITZ	NL3	G2
$s^{1/2}$	-38.6	-34.6	-39.8	-38.8
$p^{3/2}$	-34.8	-31.1	-36.3	-35.1
$p^{1/2}$	-34.6	-31.0	-36.1	-34.8
$d^{5/2}$	-29.9	-26.6	-31.4	-30.2
$d^{3/2}$	-29.2	-26.1	30.7	-29.3
$s^{1/2}$	-26.2	-23.1	-26.3	-26.1
$f^{7/2}$	-24.2	-21.3	-25.7	-24.5
$f^{5/2}$	-22.7	-20.2	-24.2	-22.8
$p^{3/2}$	-19.1	-16.5	-19.8	-19.0
$p^{1/2}$	-18.9	-16.3	-19.7	-18.7
$g^{9/2}$	-17.9	-15.3	-19.3	-18.1
$g^{7/2}$	-15.3	-13.4	-17	-15.4
$d^{5/2}$	-11.9	-9.5	-12.9	-11.6
$h^{11/2}$	-11.1	-8.8	-12.5	-11.3
$d^{3/2}$	-10.9	-8.7	-12.3	-10.7
$s^{1/2}$	-9.8	-7.2	-10.2	-9.3
$h^{9/2}$	-7.3	-6.0	-9.3	-7.5
$f^{7/2}$	-4.5	-2.4	-5.8	-4.1
$i^{13/2}$	-4.0	-2.0	-5.5	-4.1
$f^{5/2}$	-2.6	-0.9	-4.7	-2.4
$p^{3/2}$	-1.4	0.4	-2.6	-0.8

culated Δ_n for the whole atomic nuclei in the isotopic chains are displayed in FIG. 2. We obtained an arc like structure with vanishing Δ_n at $N=182, 208$ and $N=172, 184, 258$ respectively for SHF and RMF of the considered parameter sets. Further, the neutron pairing gap is found to be minimum among the isotopic chains pointing towards the magic nature of $Z=120$. Therefore, all of these force parameters are directing $Z=120$ as the next magic number after $Z=82$.

As mentioned earlier, the binding energy per particle (BE/A) is maximum for double closed nucleus compared to the neighbouring one. For example, the BE/A with SHF (FITZ set) for $^{300,302,304}120$ are 7.046, 7.048 and 7.044 MeV corresponding to $N=180, 182$ and 184 respectively. Similarly with SLy4 these values are 6.950, 6.952 and 6.933 MeV. This is reflected in the sudden jump of S_{2n} from a higher value to a lower one at the magic number in an isotopic chain. This lowering in two neutron separation energy is an acid test for shell closure investigation. FIG. 3 shows the S_{2n} as a function of neutron number for all the isotopic chain of the considered elements for both SHF and RMF formalisms. In spite of the complexity about single-particle and collective properties of the nuclear interaction some simple

Table 2: Same as Table 1. but for neutron ϵ_n (MeV).

orbit	proton (ϵ_p)			
	SLy4	FITZ	NL3	G2
$s_{1/2}$	-58.0	-50.7	-55.4	-54.0
$p_{3/2}$	-53.7	-47.4	-51.8	-50.2
$p_{1/2}$	-53.4	-47.2	-51.6	-50.0
$d_{5/2}$	-48.0	-42.9	-46.7	-45.1
$d_{3/2}$	-47.2	-42.3	-46.0	-44.3
$s_{1/2}$	-43.8	-39.2	-41.0	-40.3
$f_{7/2}$	-41.5	-37.5	-40.6	-39.1
$f_{5/2}$	-39.9	-36.4	-39.3	-37.6
$p_{3/2}$	-36.0	-32.8	-34.6	-33.5
$p_{1/2}$	-35.8	-32.5	-34.5	-33.1
$g_{9/2}$	-34.2	-31.5	-33.9	-32.5
$g_{7/2}$	-31.7	-29.6	-31.8	-30.0
$d_{5/2}$	-28.0	-26.2	-27.8	-26.3
$d_{3/2}$	-26.8	-25.2	-27.2	-25.4
$h_{11/2}$	-26.5	-25.0	-26.9	-25.3
$s_{1/2}$	-25.1	-24.1	-24.8	-23.3
$h_{9/2}$	-22.7	-22.2	-23.8	-21.8
$f_{7/2}$	-19.8	-19.2	-20.5	-18.7
$i_{13/2}$	-18.5	-18.1	-19.6	-18.1
$f_{5/2}$	-17.7	-17.5	-19.4	-16.9
$p_{3/2}$	-16.5	-15.9	-16.9	-14.9
$p_{1/2}$	-16.2	-15.7	-16.7	-14.4
$i_{11/2}$	-13.3	-14.1	-15.6	-13.1
$g_{9/2}$	-11.7	-11.9	-13.2	-11.0
$j_{15/2}$	-10.3	-10.9	-12.1	-10.5
$g_{7/2}$	-8.8	-9.6	-11.5	-8.6
$d_{5/2}$	-8.0	-8.5	-9.5	-7.2
$d_{3/2}$	-7.0	-7.7	-9.2	-6.6
$s_{1/2}$	-3.6	-5.7	-8.2	-6.0
$j_{13/2}$			-7.3	-4.3

phenomenological facts emerge from the bulk properties of the low-lying states in the even-even atomic nuclei. The S_{2n} energy is sensitive to this collective/single-particle inter play and provides sufficient information about the nuclear structure effects. From FIG. 3, we notice such effect, i.e., jump in two neutron separation energy at $N=182$ and 208 with SHF. However, such jumps are found at $N=172, 184, 258$ in RMF calculations confirming the shell closure properties of the neutron numbers.

The shell correction energy E_{shell} is a key quantity to determine the shell closure of nucleon. This concept was introduced by Strutinski [34] in liquid-drop model to take care of the shell effects. As a result, the whole scenario of liquid properties converted to shell structure which could explain the magic shell even in the frame-work of liquid-drop model. The magnitude of total (proton plus neutron) E_{shell} energy is dictated by the level density around the Fermi level. A positive E_{shell} reduces the binding energy and a negative shell correction energy increases the stability of the nucleus. As a representative case, we have depicted our SHF result of E_{shell} in FIG. 3. It is clear from the figure the extra stability of $^{302,328}120$. We find similar results of large negative shell energy for RMF calculation at neutron number $172, 184, 258$. Such calculations for few cases are reported in Ref. [35].

As a further confirmative test, the single-particle energy levels for neutrons and protons $\epsilon_{n,p}$ are analyzed. The calculated $\epsilon_{n,p}$ are presented in Table I for $^{302}120$ SHF(SLy4 and FITZ) and for $^{304}120$ RMF(NL3 and G2) as representative cases. From the Table, one can estimate the energy gaps $\Delta\epsilon_{n,p}$ for neutron and proton orbits. For example, in $^{302}120$ (FITZ), the gap $\Delta\epsilon_n = \epsilon_n(3d_{3/2}) - \epsilon_n(4s_{1/2})$ at $N=182$ is 1.977 MeV which is comparable with 1.898 MeV of the known magic gap at $N=50$ for the same nucleus. For the proton case at $Z=120$, we get $\Delta\epsilon_p = \epsilon_p(2f_{5/2}) - \epsilon_p(3p_{3/2}) = 1.340$ MeV which is again a large gap of similar order ($\Delta\epsilon_p = \epsilon_p(1g_{9/2}) - \epsilon_p(1g_{7/2}) = 1.862$ MeV) for $N=50$. Almost identical behaviour is noticed with other SHF and RMF (at $N=184$) calculations, irrespective of parameter used, confirming $Z=120$ as a clear magic number.

In summary, we have analyzed the pairing gap Δ_p and Δ_n , two-neutron separation energy S_{2n} , shell correction energy E_{shell} and single-particle energy for the whole $Z=112-130$ region covering the proton-rich to neutron-rich isotopes. To our knowledge, this is one of the first such extensive and rigorous calculation in both SHF and RMF models using a large number of parameter sets. The recently developed effective field theory motivated relativistic mean field forces G1 and G2 are also

involved. Although the results depend slightly on the forces used, the general set of magic numbers beyond ^{208}Pb are $Z=120$ and $N=172, 182/184, 208$ and 258 . The highly discussed proton magic number $Z = 114$ in the past (last four decades) is found to be feebly magic in nature.

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Abstract

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1.